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Simultaneous inverse determination of temperature-dependent thermophysical properties in fluids using the network simulation method

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Abstract

A new and efficient numerical procedure is applied for the inverse simultaneous estimation of temperature-dependent thermal properties (thermal conductivity and volumetric heat capacity) of fully developed fluids flowing through a circular duct. The procedure makes use of both the Sequential Function Specification Method as the inverse protocol and the Network Simulation Method as the numerical technique. The temperature solution at discrete times at several points of the wall–fluid interface can be solved from the direct problem by adding a normal random error. A time-dependent heat flux boundary condition is applied to the inlet region. The proposed method provides estimations of the functions, $k(T)$ and $C(T)$, regardless the waveform of those functions even without prior information on the kind of dependence concerned. The solutions take the form of piece-wise functions with a number of stretches that may be specified. A common iterative least-squares approach is used to minimize the classical functional. Various approaches to solving this problem are discussed.

Keywords: Inverse problem; Thermophysical properties; Network method

1. Introduction

Inverse problems are one of the fastest growing areas in various fields. Such problems have one main difficulty, the instability of their solution when the measurements are affected by noise, that is, their inherently ill-posed nature makes the existence of more than one solution possible. In the field of heat transfer, different types of inverse heat conduction problem (IHCP) exist, such as: (i) the identification of thermophysical properties, (ii) the determination of boundary conditions, (iii) the determination of heat generation sources and (iv) determination of the initial state.

If the thermal properties are dependent on the temperature, the problems are non-linear and the solution of both the direct (DHCP) and inverse (IHCP) problems generally require the use of approximate numerical techniques, such as finite-differences, finite-element, enthalpy formulation or Laplace transformations, among others, Beck et al. [1, pp. 78–102]. The greater the temperature dependence of k and/or C, the more difficult it is to reach a convergence solution for either DHCP or IHCP.

Within the first type (for solids) of IHCP, those concerned with the estimation of temperature-dependent thermophysical properties may be particularly complex, especially if no information is available on the functional form of the unknown property and if the functional minimization has to be performed in an infinite dimensional space of functions. Hence, the simultaneous estimation of both k and C in one sole experiment, a recent problem that

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Nomenclature

C	volumetric heat capacity (J m^{-3} K^{-1}), $C = \rho c_e$
C	capacitor and capacitance (F), Fig. 1
c_e	specific heat (J kg^{-1} K^{-1})
D_1, D_2	length parameters
E	voltage-control voltage-source, Fig. 1
F	functional defined in Eq. (5)
G	voltage-control current-source, Fig. 1
j	heat flux rate (W m^{-2})
k	thermal conductivity (W m^{-1} K^{-1})
K_k, K_C	increment of the unknown quantities k and C, respectively
L	length of the duct
N_x	number of volume elements in the axial direction
N_r	number of volume elements in the radial direction
m, n	integer numbers
P	number of sensors
R	inner radius of the duct (m)
R	resistor (Ω)
r	spatial radial co-ordinate (m)
t	time (s)
T	temperature (°C)
u	axial fluid velocity (m s^{-1})
x	spatial axial co-ordinate (m)
z	$1, 2, \ldots, Z$
Z	number of stretches of the piece-wise function in the IHCP
Δt	interval of time between measurements
ΔT_a	temperature interval associated to the functional
Δr	thickness of the control volume in the radial direction
Δx	thickness of the control volume in the axial direction
∞	relate to a very high value

Greek symbols

δ_k, δ_C	convergence parameters for k and C, respectively
ε	normal random error
ζ	thickness of the wall
λ_k, λ_C	reduction factors for k and C, respectively
ρ	density (kg m^{-3})
σ	standard deviation of the errors in the temperature measurements
ω	random number variable
ψ	integer number, $1, 2, 3, \ldots$

Subscripts

end	reference to the final temperature of the range of estimation selected
ex	exact solution of the direct problem
f	fluid, also a particular location for measurements, $1, 2, \ldots, P$
g	number of temperatures for each sensor and each $\Delta T_a, 1, 2, \ldots$
i, j	denote the location of the cell, Fig. 1
ini	reference to the initial temperature of the range of estimation selected
mea	measurement provide by the sensor
0	initial condition
obt	solution of the inverse problem
s	solid

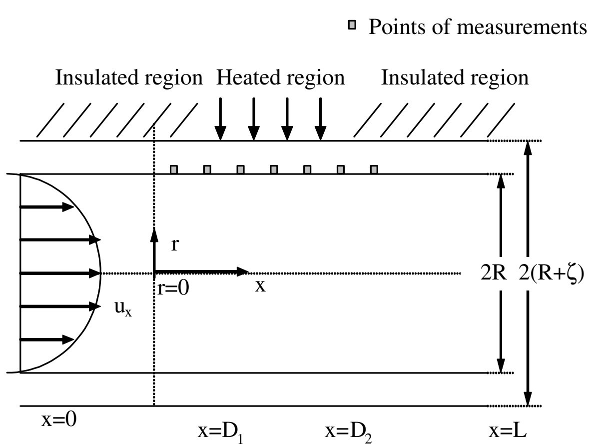

Fig. 1. Physical scheme of the problem.

has scarcely been treated, Yang [2], is much more difficult to solve. A convergent and valid estimation can be reached by iteration, taking the temperature measurements (DHCP) at several points of the solid and defining an adequate functional that compares step by step the temperatures provided by the inverse solution and the measurements.

Huang and Özisik [3] obtained precise estimations for 1-D solids in the case of linear and sinusoidal dependencies of both properties using approximate direct integration method. Huang and Yan [4] also obtained estimations of the same dependencies utilizing the conjugate gradient method of minimization and the adjoint equation while Sawaf et al. [5] also obtained estimations of these properties in a 2-D orthotropic solid using Levenberg–Marquardt's iterative procedure. Using the easier function estimation technique, which assumes that the form of the function is known, Dantas and Orlande [6] made simultaneous estimations of k_f and C_f applying the conjugate gradient method; including a study of the influence of sensor location in the same paper.

In this work, a new procedure is developed for the simultaneous determination of the temperature-dependent thermal properties (thermal conductivity and volumetric heat capacity) of fluids flowing through circular ducts of finite thickness. This is a conjugate conduction–convection non-lineal problem, in which a fully developed laminar flow, assuming bi-dimensional (axial–radial) wall and fluid heat conduction, is considered, with negligible viscous dissipation.

Huang and Özisik [7] used a similar mathematical model for estimating the space wise variation of an unknown applied wall heat flux in laminar flows by means of the conjugate gradient method. The authors did not consider the wall effect (negligible thickness of the duct) and assumed constant thermal properties. Few works have been devoted to the estimation of physical parameters in fluids. Oul-Lahoucine et al. [8] developed an experimental method using a thermistor probe assisted by numerical analysis to measure the constant thermal conductivity of liquids and powders using the least square method. Liu and Özisik [9] determined the constant thermal conductivity and volumetric heat capacity, simultaneously, of a fluid in laminar forced convection inside a circular duct, using the iterative algorithm of Levenberg–Marquardt's method. Finally, Kim and Lee [10] developed an inverse method for simultaneously estimating the temperature-dependent thermophysical properties in fluids based on experimental methodology and the algorithm of Levenberg–Marquardt's method, but with prior knowledge of the type of dependence.

This general propose procedure uses the network simulation method (NSM hereafter) as the numerical technique and does not require prior information on the kind of dependence of the thermal conductivity and heat capacity. The procedure may be applied regardless of the kind of temperature dependence of these properties, as long as they are continuous temperature functions given by an explicit mathematical function or by finite stretches piece-wise functions. The estimations of k_f and C_f are piece-wise functions with as high a number of stretches as is required to approximate the inverse solutions to the exact values.

The typical functional of these problems contains the simulated measurement data taken at various points of the system. These measurements are then compared within the functional with the solution of the partial inverse problem by applying NSM [11] in each iteration, in order to estimate the stretches that conform the piece-wise final solution. NSM has already been successfully applied to solving several types of direct problem: see Alhama and González-Fernández [12] for applications to food processing and Zueco et al. [13] for applications to the laminar flow of fluids in ducts. As regards inverse problems, NSM was applied by Zueco et al. [14] and Alhama et al. [15] to estimate temperature-dependent thermal properties, by Alhama et al. [16] for unsteady heat flux wall estimations and by Zueco et al. [17] for estimating the time-dependent heat transfer coefficient, while Zueco and Alhama [18] developed an iterative algorithm for the estimation of the temperature-dependent emissivity of solid metals. Among the advantages of the NSM is the fact that no mathematical manipulations or convergence criteria are needed to solve the finite difference equations resulting from the discretization of the partial difference equations of the mathematical model. Both tasks are carried out by the powerful software PSPICE [19] used to solve the network model.

2. Direct problem: the system and governing equations

Cylindrical 2-D geometry is used. A fully developed flow fluid that crosses a duct of length L, radius R and thickness E, and whose initial temperature (duct and fluid) is T_0, is suddenly submitted to heating process by applying a time-variable heat flux over the inlet region $D_1 \leqslant x \leqslant D_2$. The rest of the duct region is under adiabatic condition. Both the thermal conductivity and heat capacity of the fluid depend on temperature, $k_f(T)$ and $C_f(T)$, while the same properties for the duct are constant. The mathematical model of the DHCP is defined by the following set of equations:

$$(1/r)(\partial[(rk_s)\partial T_s/\partial r]/\partial r) + (\partial[k_s\partial T_s/\partial x]/\partial x) = (\rho c_e)_s \partial T_s/\partial t,\quad \text{at } R < r < R+E \text{ and } t > 0 \tag{1a}$$

$$(1/r)(\partial[(rk_f)\partial T_f/\partial r]/\partial r) + (\partial[k_f\partial T_f/\partial x]/\partial x) = (\rho u_x c_e)_f \partial T_f/\partial x + (\rho c_e)_f \partial T_f/\partial t,\quad \text{at } 0 < r < R \text{ and } t > 0 \tag{1b}$$

Boundary conditions:

$$T_f = T_s = T_0,\quad \text{at } x = 0,\ t > 0 \tag{2a}$$

$$\partial T_s/\partial r = \partial T_f/\partial r = 0 \quad \text{at } x = L,\ 0 < r < R+E,\ t > 0 \tag{2b}$$

$$\partial T_f/\partial r = 0 \quad \text{at } r = 0,\ 0 < x < L,\ t > 0 \tag{2c}$$

$$T_s = T_f;\ k_s\partial T_s/\partial r = k_f\partial T_f/\partial r \quad \text{at } r = R,\ 0 < x < L,\ t > 0 \tag{2d}$$

$$-k_s\partial T_s/\partial r = q_s \quad \text{at } r = R+E,\ D_2 > x > D_1,\ t > 0 \tag{2e}$$

$$\partial T_s/\partial r = 0 \quad \text{at } r = R+E,\ 0 < x < D_1,\ D_2 < x < L,\ t > 0 \tag{2f}$$

Initial condition:

$$T_f = T_s = T_0 \quad \forall r, x,\ t = 0 \tag{3}$$

Eqs. (1a) and (1b) refer to the wall and fluid regions, respectively. $u_x = 2u_{av}[1 - (r/R)^2]$, with u_{av} being the average velocity.

In the case of the direct problem considered above, the thermal property dependencies are regarded as known:

$$k_f(T_\psi) = k_\psi,\quad 1 \leqslant \psi \leqslant m \tag{4a}$$

$$C_f(T_\psi) = C_\psi,\quad 1 \leqslant \psi \leqslant n \tag{4b}$$

where k_f and C_f are the thermal conductivity and the volumetric heat capacity of the fluid, respectively; x, r and t are the independent variables, axial position, radial position and time, q_s is the constant incident heat flux applied to the wall and m and n are integer numbers. Piecewise dependencies (4a) and (4b) are able to reproduce any kind of function, $k_f(T_\psi)$ or $C_f(T_\psi)$, including the strong dependencies appearing in phase change phenomena, providing m

and n are large enough. Arbitrary mathematical functions for $k_f(T)$ and $C_f(T)$ may also be assumed by the method.

3. The inverse problem

In the inverse problem case the temperature dependencies of the thermal properties are unknown and they must be predicted from the temperature measurements taken at P different positions (number of sensors) x_f, $f = 1, \ldots .P$, at the wall–fluid interface ($r = R$).

The performance function for the identification of $k_f(T)$ and $C_f(T)$ is expressed by the sum of square residuals between the estimated and measurement temperatures as follows:

$$F[k_f, C_f] = \sum_{f=1}^{P} \left[\sum_{g=0}^{r_{z,f}} T_{\text{obt},g}(x_f, t_g, R, k, C) - T_{\text{mea},g}(x_f, t_g, R, \varepsilon) \right]^2 \tag{5}$$

with $z = 1, 2, \ldots, Z$ the actual stretch (Z being the number of stretches of the piece-wise function), $T_{\text{obt}}(x_f, t_g, r = R, k, C)$ the estimated temperature and $T_{\text{mea}}(x_f, t_g, r = R, \varepsilon)$ the measured temperature at the localization ($x = x_f$, $r = R$). The number of terms within the functional for each stretch and each sensor, $r_{z,f}$, coincides with the total number of measurement for each sensor within the interval of estimation defined as $T_0 + (z-1)/2\ \Delta T_a \to T_0 + (z+1)/2\Delta T_a$. For example, for $T_0 = 0\ °C$, $\Delta T_a = 20\ °C$ and $z = 4$, the interval is $30\ °C \to 50\ °C$, Fig. 3. Δt is the time interval between measurements. The set $T_{\text{mea}}(x_f, t_g, R, \varepsilon)$ is obtained by adding a normal random error, $T_{\text{ex}}(x_f, t_g, R)$, to the solution of the DHCP.

$$T_{\text{mea}}(x_f, t_g, R, \varepsilon_g) = T_{\text{ex}}(x_f, t_g, R) + \varepsilon_g \tag{6}$$

where $\varepsilon_g = \omega_g \sigma$, ω_g is a normally distributed random number, with zero mean, a standard deviation of unity and a 99% confidence level. ω_g lies within the range $-2.576 < \omega_g < 2.576$, with σ being the standard deviation of the errors in the temperature measurements.

The value of Z is related to the temperatures by the expression

$$Z = 2(T_{\text{end}} - T_{\text{ini}})/\Delta T_a \tag{7}$$

where T_{end} is the final temperature of the range to be estimated.

A common iterative least-squares approach is used to minimize the performance function F, which leads to the solution of the IHCP in the form of a linear piece-wise function of Z stretches of variable slope and temperature interval ΔT_a. A program routine based in the Sequential Function Specification Method (SFSM) [1, pp. 119–133], allows the slope of the actual stretch to be continuously changed by adding new steps to both the volumetric heat capacity and the thermal conductivity, until the functional reaches a minimum value. This routine runs iteratively until a prefixed convergence criterion is satisfied. The numerical resolution of the energy equation is solved with the Network Simulation Method.

The formulation of the IHCP involves equations (1), (2a–f), (3) and the set of temperatures $T_{\text{mea}}(x_f, R, t_g, \varepsilon_g)$ from Eq. (6). With this information available, the aim now is to estimate simultaneously the temperature dependence of $k_f(T)$ and $C_f(T)$ for the fluid, whose exact values are given by Eqs. (4a) and (4b), by means of new piece-wise function of m and n stretches, respectively.

4. Network model

It is known [13] that with larger values of the Péclet number ($2u_{av}RC_f/k_f$), both the fluid axial conduction and the effect of wall conduction on heat transfer are negligible. Nevertheless, the proposed model does not assume this simplified hypothesis and, consequently, may be applied to a wide range of Péclet numbers. In [13] we explain the steps for designing the network model of the direct problem for the whole medium (wall and fluid) when thermal properties are constant. The same model is used here for the wall. As regards the fluid network model, the resistors of the network model (for the fluid) were changed by voltage-control current-sources due to the temperature-dependence of their thermal properties.

The geometry of each volume element is a circular crown, whose surface section has a dimension of Δr and Δx, which are the number of cells in the radial and axial directions of $N_{r,w}$ (wall), $N_{r,f}$ (fluid) and N_x, respectively. Spatial discretization of Eq. (1b) leads to the following ordinary differential equation:

$$\begin{aligned}
&[T_{i,j-\Delta r/2} - T_{i,j}]/[2\Delta x r_{i,j-\Delta r/2}/\Delta r]k_{i,j-\Delta r/2}(T_{i,j-\Delta r/2}) \\
&\quad - [T_{i,j} - T_{i,j+\Delta r/2}]/[2\Delta x r_{i,j+\Delta r/2}/\Delta r]k_{f,i,j+\Delta r/2}(T_{i,j+\Delta r/2}) \\
&\quad + [T_{i-\Delta x/2,j} - T_{i,j}]/[2k_{f,i-\Delta x/2,j}(T_{i-\Delta x/2,j})r_{i,j}\Delta r/\Delta x] \\
&\quad - [T_{i,j} - T_{i+\Delta x/2,j}]/[2k_{f,i+\Delta x/2,j}(T_{i+\Delta x/2,j})r_{i,j}\Delta r/\Delta x] \\
&\quad = 2u_{av}(1 - r_{i,j}^2/R^2)[T_{i+\Delta x/2,j} - T_{i-\Delta x/2,j}]r_{i,j}\Delta r \\
&\qquad + C_{f,i,j}(T_{i,j})r_{i,j}\Delta r\Delta x\,\mathrm{d}T_i/\mathrm{d}t
\end{aligned} \tag{8}$$

where $T_{i,j}$ is the temperature in the centre, $T_{i,j+\Delta r/2}$, $T_{i,j-\Delta r/2}$ are the temperatures in the radial extremes, and $T_{i+\Delta x/2,j}$, $T_{i-\Delta x/2,j}$ are the temperatures in the axial extremes (Fig. 2). Defining the currents

$$j_{i,j-\Delta r/2} = [T_{i,j-\Delta r/2} - T_{i,j}]/[2k_{f,i,j-\Delta r/2}(T_{i,j-\Delta r/2})r_{i,j-\Delta r/2}\Delta x/\Delta r] \tag{9a}$$

$$j_{i,j+\Delta r/2} = [T_{i,j} - T_{i,j+\Delta r/2}]/[2k_{f,i,j+\Delta r/2}(T_{i,j+\Delta r/2})r_{i,j+\Delta r/2}\Delta x/\Delta r] \tag{9b}$$

$$j_{i-\Delta x/2,j} = [T_{i-\Delta x/2,j} - T_{i,j}]/[2k_{f,i-\Delta x/2,j}(T_{i-\Delta x/2,j})r_{i,j}\Delta r/\Delta x] \tag{9c}$$

$$j_{i+\Delta x/2,j} = [T_{i,j} - T_{i+\Delta x/2,j}]/[2k_{f,i+\Delta x/2,j}(T_{i+\Delta x/2,j})r_{i,j}\Delta r/\Delta x] \tag{9d}$$

$$j_{i,j,x} = 2u_{av}(1 - r_{i,j}^2/R^2)C_{f,i,j}(T_{i,j})[T_{i+\Delta x/2,j} - T_{i-\Delta x/2,j}]r_{i,j}\Delta r \tag{9e}$$

$$j_{i,j,1} = C_{f,i,j,av}r_{i,j}\Delta r\Delta x\,\mathrm{d}T_{i,j}/\mathrm{d}t \tag{9f}$$

$$j_{i,j,2} = \Delta C_{f,i,j}(T_{i,j})r_{i,j}\Delta r\Delta x\,\mathrm{d}T_{i,j}/\mathrm{d}t \tag{9g}$$

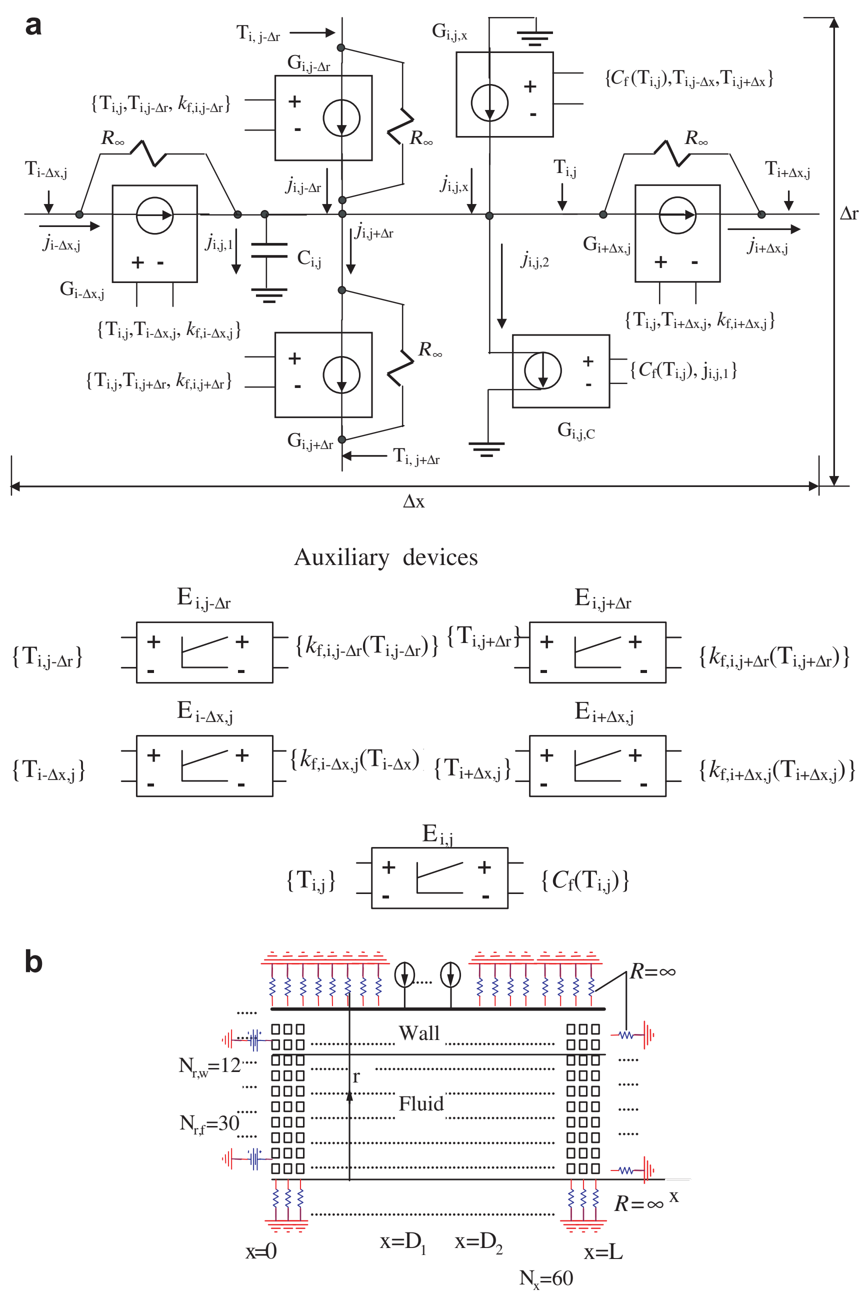

Fig. 2. Network model. (a) For the finite volume element. (b) For the boundary conditions.

Eq. (8) may be written as Kirchhoff's law for the following currents:

$$j_{i,j-\Delta r/2} - j_{i,j+\Delta r/2} + j_{i-\Delta x/2,j} - j_{,i+\Delta x/2,j}$$
$$= j_{i,j,x} + j_{i,j,1} + j_{i,j,2} \tag{10}$$

The network model for the cell is now designed (Fig. 2a). Eqs. (9a) and (9b) define the current that leaves and enters the 2-D cell i,j in the radial direction; $j_{i,j\pm\Delta r} = \pm(T_{i,j} - T_{i,j\pm\Delta r})(\Delta r/2k_{\mathrm{f},i,j\pm\Delta r/2}r_{i,j\pm\Delta r/2}\Delta x)$ being $k_{\mathrm{f},i,j\pm\Delta r}$, the conductivity to both ends of the volume element (cell) i,j, dependent on temperatures $T_{i,j\pm\Delta r}$. Heat fluxes $j_{i,j\pm\Delta r}$ are implemented by means of voltage-control current-sources ($G_{i,j-\Delta r}$ and $G_{i,j+\Delta r}$), whose value, specified by Eqs. (9a) and (9b), can be programmed. Two control voltage sources, $E_{i,j-\Delta r}$ and $E_{i,j+\Delta r}$, controlled by $T_{i,j-\Delta r}$ and $T_{i,j+\Delta r}$, respectively, provide the values of $k_{i,j-\Delta r}$ and $k_{i,j+\Delta r}$. To implement Eqs. (9c)–(9e), which are related to axial direction, the same kinds of devices are used. The volumetric heat capacity of the cell, $C_{\mathrm{f},i,j}(T_{i,j})$, is evaluated by means of Eq. (4b). The value of $C_{\mathrm{f},i,j}$ is divided into two parts in the form $C_{\mathrm{f},i,j,} = C_{\mathrm{f},i,j,\mathrm{av}} + \Delta C_{\mathrm{f},i,j}$, with $C_{\mathrm{f},i,j,\mathrm{av}}$ being the mean value of $C_{\mathrm{f},i,j}$ within the whole temperature estimation range, so that it is possible to implement the initial condition by fixing the voltage of the capacitor. Eq. (9f) defines a capacitor, $C_{i,j}$, of capacitance $C_{i,j} = C_{\mathrm{f},i,j,\mathrm{av}}r_{i,j}\,\Delta r\Delta x$, while Eq. (9g) is implemented by a voltage-control current-generator, $G_{i,j,C}$ which is controlled by both $E_{i,j}$ and the current of $C_{i,j}(j_{i,j,1})$, according to the function "$j_{i,j,1}[(C_{f,i,j} - C_{f,i,j,av})/C_{f,i,j,av}]$". The network is completed by adding R_∞, a high value resistor needed to provide continuity in the circuit, according to PSPICE requirements.

Each cell is 2-D electrically connected to adjacent cells to form the whole model of the medium (Fig. 2b). A resistor of infinite value, R_∞, is used to implement the adiabatic boundary conditions (2b), (2c) and (2f), while constant voltage sources are implemented to satisfy condition (2a). Heat flux sources are also implemented in the model for the boundary condition (2e). As regards boundary condition (2d), this is assumed by simple connection of the fluid and wall cells. Finally, initial condition (3) is implemented by charging the capacitors of each cell to a voltage value equal to the initial temperature.

5. The present procedure

The algorithm for the iterative scheme (Visual C++ program) is developed from the algorithm proposed by Zueco [20]. Each segment of the piece-wise function runs from the temperature $T_0 + ((z-1)/2)\Delta T_a$ to the temperature $T_0 + ((z+1)/2)\Delta T_a$. A special auxiliary device, named E in Fig. 2a, generates the piece-wise temperature-dependent function both for the conductivity and the heat capacity. This device makes it possible to select both the length of each stretch and the change in its slope, which continuously increases or decreases until the convergence criteria are satisfied. Fig. 3 shows an example for $z = 4$.

The following steps summarize the proposed procedure:

(i) Solve DHCP numerically (using the NSM); from this solution, the set $T_{mea}(x_f, t_g, R, \varepsilon_g)$ is obtained using a normal random number generator. Fix the values of K_k and K_C. $T_0 = T_{ini}$.

(ii) Estimate the initial points of the first stretch of the two estimations (conductivity and heat capacity), i.e. $(k_{f,ini}, T_{f,ini})$ and $(C_{f,ini}, T_{f,ini})$. To this end, choose a horizontal stretch of length $\Delta T_{ini} < \Delta T_a$ that minimizes the functional equation (5), $z = 0$.

(iii) Fix the values of ΔT_a (temperature interval of the stretches), δ_k and δ_C (convergence criteria) and $\lambda_k = \lambda_C = 0.75$ (reduction factors for the slopes). Calculate Z (total number of stretches).

(iv) $z = z + 1$, $T_z = T_{z-1} + \Delta T_a/2$, $T_{z+1} = T_{z-1} + \Delta T_a$. Evaluate the values of $r_{z,f}$.

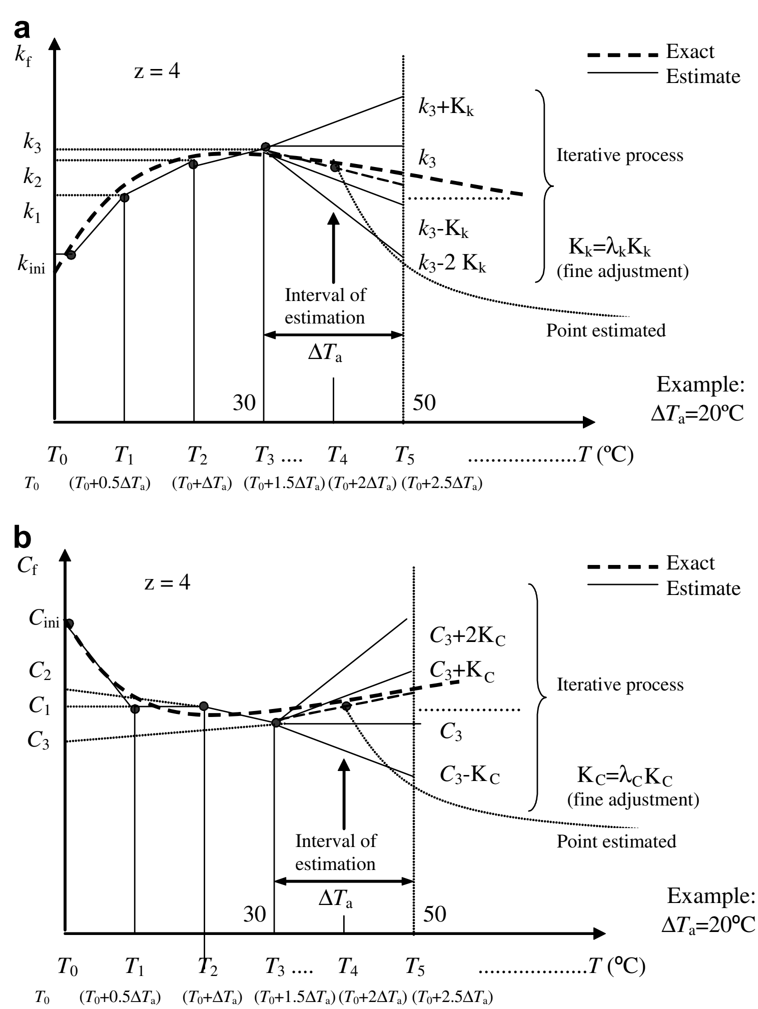

Fig. 3. Illustration of temperature-dependent function estimation for both functions for $z = 4$. (a) Thermal conductivity. (b) Volumetric heat capacity.

(v) Determine F, Eq. (5), for each pair of segments, one belonging to the (three) segments limited by the points $(k_{f,z-1}, T_{z-1})$ and $(k_{f,z-1} + K_k, T_{z+1})$, $(k_{f,z-1}, T_{z-1})$ and $(k_{f,z+1}, T_{z+1})$, and $(k_{f,z-1}, T_{z-1})$ and $(k_{f,z-1} - K_k, T_{z+1})$, and the other belonging to the (three) segments limited by the points $(C_{f,z-1}, T_{z-1})$ and $(C_{f,z-1} + K_C, T_{z+1})$, $(C_{f,z-1}, T_{z-1})$ and $(C_{f,z+1}, T_{z+1})$, and $(C_{f,z-1}, T_{z-1})$ and $(C_{f,z-1} - K_C, T_{z+1})$. The set of $T_{obt}(x_f, t_g, R, k, C)$ within the functional is evaluated by applying NSM as a direct problem using the $k_f(T)$ and $C_f(T)$ values from the segments defined above. The final points that belongs to the segments connected to the minimum value of the functional, $k_{f,z-1} = k_{f,z+1,min}$ and $C_{f,z+1} = C_{f,z+1,min}$ are retained.

(vi) $K_k = \lambda_k K_k$ and $K_C = \lambda_C K_C$ (fine adjustment).

(vii) Repeat steps (v) and (vi) until $K_k < \delta_k$ and $K_C < \delta_C$, then retaining the points $(k_{f,z+1,min}, T_{z+1})$ and $(C_{f,z+1,min}, T_{z+1})$.

(viii) Select the extreme of the actual stretch, $(k_{f,z}, T_z) = [(k_{f,z-1}, T_{z-1}) + (k_{f,z+1,min}, T_{z+1})]/2, (C_{f,z}, T_z) = [(C_{f,z-1}, T_{z-1}) + (C_{f,z+1,min}, T_{z+1})]/2$.

(ix) If $z < Z$, go to step (iv) to estimate the new stretch.

(x) If $z = Z$, solve the direct problem with the complete inverse solution of $k_f(T)$ and $C_f(T)$ for the total duration of the transitory, and evaluate the complete functional F corresponding to this estimation.

(xi) Repeat steps (ii)–(x) for new, other values (lower and higher) of K_k and K_C (the criteria for determining these new values of K_k and K_C is open at the program, tentative tests can help to establish this criteria) and compare, successively, the actual complete functional (for the whole temperature-range of the estimated dependences of k_f and C_f) with the former until the actual becomes smaller. Finally, retain the final estimation (the solution of the inverse problem) as the one corresponding to the minimum complete functional.

As a general rule, the points where measurements are taken must be selected both near and within the region $D_1 < x < D_2$ (heated region), since this is where the greatest changes in temperature occur. It is convenient to include measurement points in the adiabatic region in order to improve the sensitivity of the estimation. On the other hand, the selection of parameters q_s, ΔT_a, Δt and the number of temperature measurements must be chosen so that $r_{z,f}$ is in the order of 20 measurements or larger. For the examples solved here, five (three in the heated region and two in the adiabatic region) and seven (three in the heated region and four in the adiabatic region) regularly distributed measurement points were selected, Fig. 1. $q_s = 0.5\ \mathrm{W\ m^{-2}}$, $\Delta T_a = 20$ and 30 °C and $\Delta t = 0.5$ s.

6. Results and discussion

Figs. 4–10 show the estimations provided by the proposed method for different values assigned to the parameters P, σ, Δt, ΔT_a, K_k and K_C. Exact solutions is also included in the figures (continuous line). The value of u_{av} for all the figures is 0.2 m/s, the radius and the length of the duct are 0.2 m and 1.5 m, respectively, the thickness

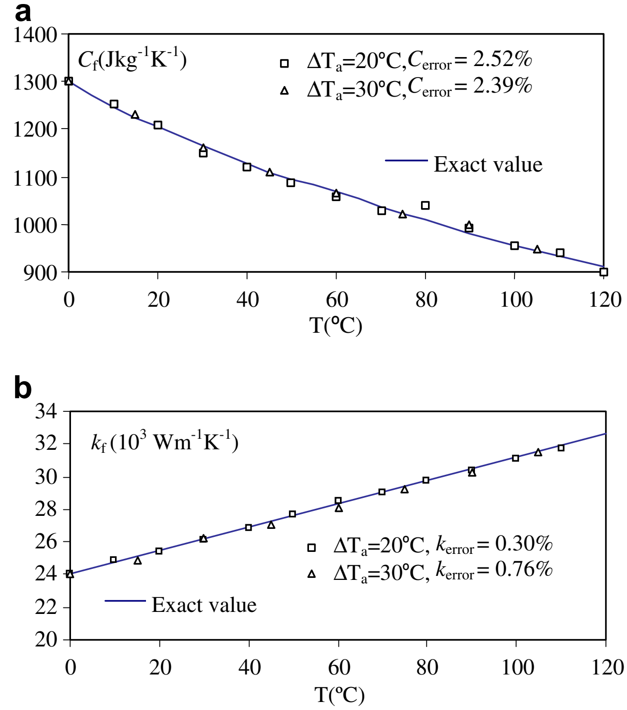

Fig. 4. $k_f(T)$ and $C_f(T)$ estimations with $P = 5$, $\sigma = 0$, $\Delta t = 0.5$ s, $K_k = 1\mathrm{E}{-03}$, $K_C = 30$, $\Delta T_a = 20$ and 30 °C.

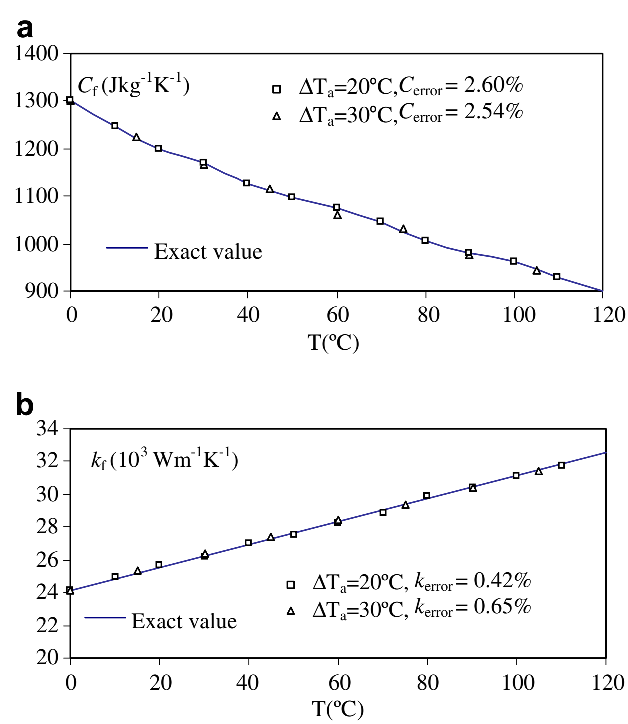

Fig. 5. $k_f(T)$ and $C_f(T)$ estimations with $P = 7$, $\sigma = 0.5$, $\Delta t = 0.5$ s, $K_k = 1\mathrm{E}{-03}$, $K_C = 30$, $\Delta T_a = 20$ and 30 °C.

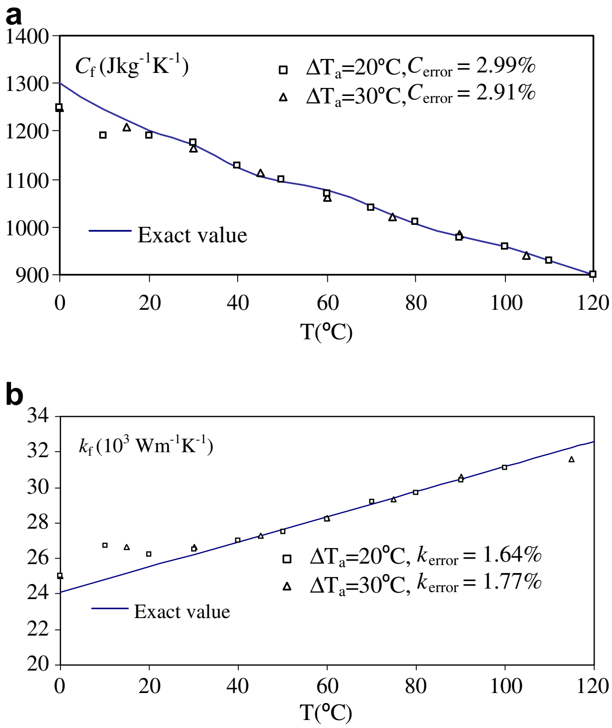

Fig. 6. $k_f(T)$ and $C_f(T)$ estimations with $P = 7$, $\sigma = 0.5$, $\Delta t = 0.5$ s, $K_k = 1\text{E}{-}03$, $K_C = 30$, $\Delta T_a = 20$ and 30 °C, and different initial estimations.

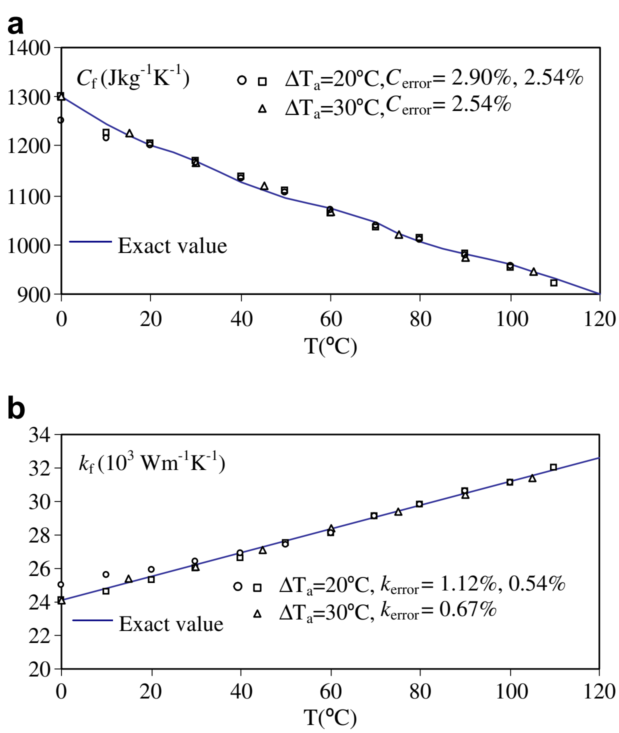

Fig. 7. $k_f(T)$ and $C_f(T)$ estimations with $P = 7$, $\sigma = 1.0$, $\Delta t = 0.5$ s, $K_k = 1\text{E}{-}03$, $K_C = 30$, $\Delta T_a = 20$ and 30 °C, and different initial estimations.

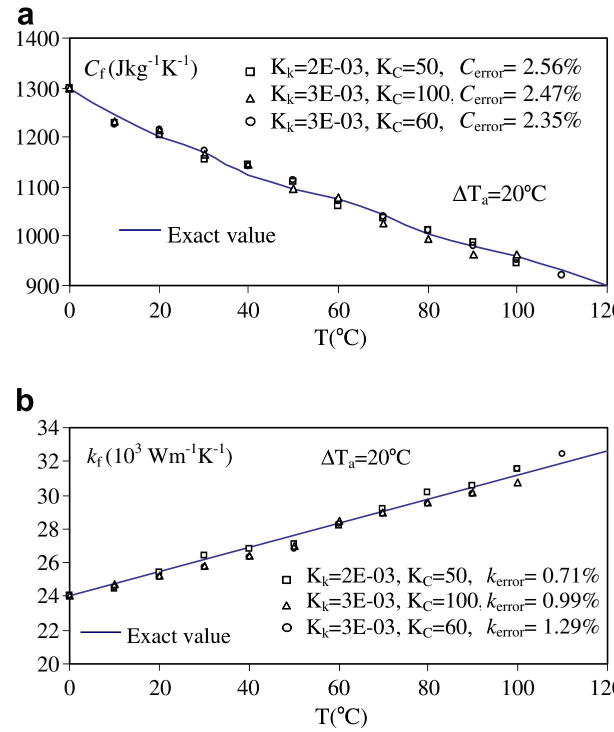

Fig. 8. $k_f(T)$ and $C_f(T)$ estimations with $P = 7$, $\sigma = 1.0$, $\Delta t = 0.5$ s, $\Delta T_a = 20$ °C, $K_k = 2\text{E}{-}03$ and $3\text{E}{-}03$, $K_C = 50$, 60 and 100.

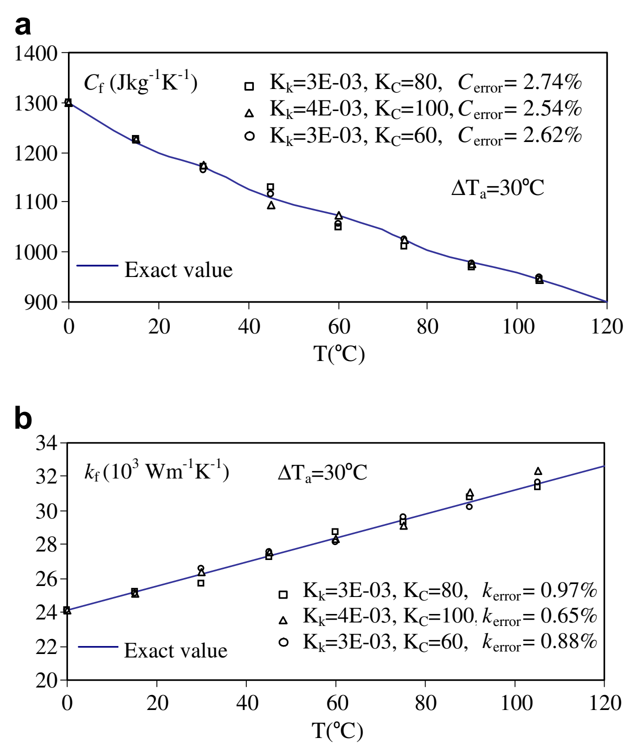

Fig. 9. $k_f(T)$ and $C_f(T)$ estimations with $P = 7$, $\sigma = 1.0$, $\Delta t = 0.5$ s, $\Delta T_a = 30$ °C, $K_k = 3\text{E}{-}03$ and $4\text{E}{-}03$ and $K_C = 60$, 80 and 100.

of the wall 0.02 m, the thermal diffusivity of the wall is $1.5\text{E}{-}05\ \text{m}^2\,\text{s}^{-1}$, while the incident heat flux applied to the wall is $q_s = 0.5\ \text{W m}^{-2}$. The exact values of the thermal properties are specified in Eq. (4a) for the thermal conductivity, $k_f(T_\psi) = k_\psi$ for $1 \leqslant \psi \leqslant m = 2$, where $k_f(0\ °\text{C}) = 24.08\text{E}{-}03\ \text{W m}^{-1}\,\text{K}^{-1}$ and $k_f(120\ °\text{C}) = 32.61\text{E}{-}03\ \text{W m}^{-1}$

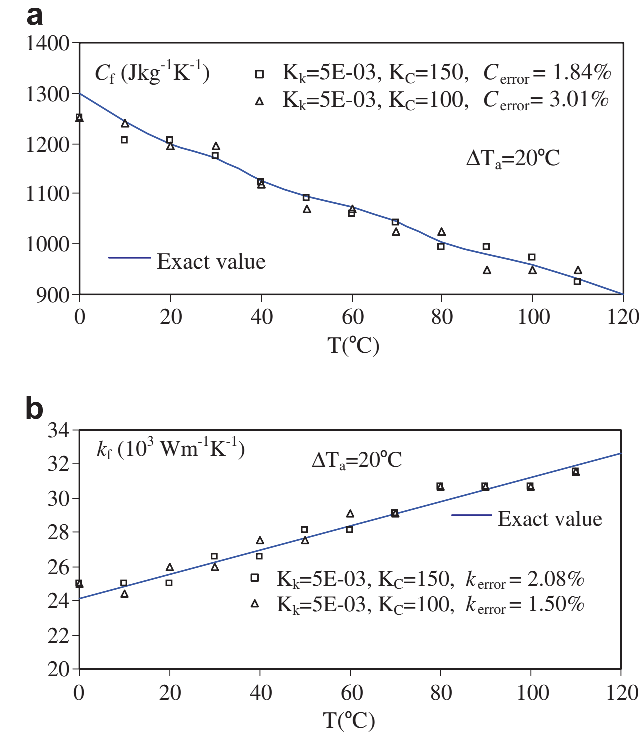

Fig. 10. $k_f(T)$ and $C_f(T)$ estimations with $P = 7$, $\sigma = 1.0$, $\Delta t = 0.5$ s, $\Delta T_a = 20$ °C, $K_k = 5\text{E}{-03}$ and $K_C = 100$ and 150.

K^{-1} and in Eq. (4b) for the volumetric capacity heat, $C_f(T_\psi) = C_\psi$ for $1 \leqslant \psi \leqslant n = 7$, where $C_f(0\ °C) = 1299.60$ J m^{-3} K^{-1}, $C_f(20\ °C) = 1202.17$ J m^{-3} K^{-1}, $C_f(40\ °C) = 1169.70$ J m^{-3} K^{-1}, $C_f(60\ °C) = 1104.75$ J m^{-3} K^{-1}, $C_f(80\ °C) = 1039.80$ J m^{-3} K^{-1}, $C_f(100\ °C) = 974.85$ J m^{-3} K^{-1}, $C_f(120\ °C) = 909.90$ J m^{-3} K^{-1}.

For five measurement points and assuming no error in the temperatures, Fig. 4 shows the estimations for heat capacity and thermal conductivity of the fluid, a and b, respectively. If there is a small error in the measurements ($\sigma = 0.5$) and if the same values for the rest of parameters are used, similar results are obtained by increasing the number of measurement points from 5 to 7. When $\sigma = 0.5$, the measured temperature errors will be within -1.288 to 1.288 for a 99% confidence bounds, which implies that a total of about 2.576 temperature errors is allowed. The temperature at x_f will range from 0 to 120 °C, thus the average relative measurement error is about 4%. For the case when $\sigma = 1.0$ the average relative measurement error is about 6.6%.

The average relative error between the exact and estimated values for k_f and C_f is defined as

$$k_{error} = \sum_{j=1}^{Z} \left| \frac{k_{f,ex}(x_f, t_j) - k_{f,obt}(x_f, t_j)}{k_{f,ex}(x_f, t_j)} \right| \frac{100\%}{Z} \tag{11}$$

$$C_{error} = \sum_{j=1}^{Z} \left| \frac{C_{f,ex}(x_f, t_j) - C_{f,obt}(x_f, t_j)}{C_{f,ex}(x_f, t_j)} \right| \frac{100\%}{Z} \tag{12}$$

Table 1
Minimum functional and average relative errors for different values of K_k and K_C for $P = 7$, $\sigma = 1.0$, Δt =0.5 s and ΔT_a =20 °C

K_k	K_C	Functional	C_{error} (%)	k_{error} (%)
1.0E−03	30	666E+03	2.54	0.544
2.0E−03	50	804E+03	2.56	0.713
5.0E−03	100	1627E+03	2.87	1.12

The values of these errors are shown in each simulation (Figs. 4–10). Errors are always less than 3%, a value quite acceptable for this kind of inverse problems that proves the robustness of the proposed method. Also, errors increase as σ increases and P decreases as expected.

To appreciate the effect of the value of the initial point of the estimation, different points have been selected, eliminating step (ii) of the procedure, providing the estimations of Figs. 6 and 7, for which $\sigma = 0.5$ and 1, respectively. As may be seen, this effect can only be appreciated at the two or three first points of the estimation. Also, it can be seen that a decrease in the value of ΔT_a leads to increase the errors in the first points of estimations. As expected, the estimations of Fig. 6, past the first two stretches, are more exact than those in Fig. 7.

Figs. 8–10 are devoted to a study of the influence of the parameters K_k and K_C (increments of the unknown quantities thermal conductivity and heat capacity, respectively), using two values for ΔT_a. Due to the large number of variables that influences the problem it is not easy to infer the effect of each one in the error. Finally Table 1 shows the values of the average relative errors for three simulations with $P = 7$, $\sigma = 1.0$, Δt =0.5 s and ΔT_a =20 °C. The value of the functional, assuming that $r_{z,f}$ includes the total number of measurement for each sensor within the complete interval of 0–120 °C, is also shown in the table. An increase in the functional provides an increase in the average relative error. The minimum value of the functional (and error) occurs for $K_k = 1.0\text{E}{-03}$ and $K_C = 30$ (Fig. 7), so that this is the best estimation obtained by the method for this problem.

7. Conclusion

A procedure for the simultaneous estimation of thermal conductivity and heat capacity for a fluid flowing through a circular duct in the form of an inverse problem is proposed. Standard, easy to implement boundary conditions are used. Several examples are solved, showing that it is necessary to install several measurement points both near and within the region where the incident heat flux is applied. The influence of all the typical parameters that take part in this kind of problems, such as the number of reading points, the error in the temperature measurements, the temperature range for each stretch, the initial point of estimation and the initial increases in the slopes of the stretches of each property. Final estimations, in general, closely agree with the exact solution.

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